

Gradient-based guidance for controlling performance in early design exploration

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Abstract

In recent years, heuristic optimization techniques have become widely used in architectural and structural design practice. However, when used as a black box solver rather than an interactive design tool, such strategies become essentially passive, ceding considerable design control to the computer. Although researchers have developed interactive versions of common approaches such as evolutionary algorithms, there has been less focus on finding ways to use gradient-based optimization techniques in cases where designers seek increased creative control. This paper explores two ways in which gradient-based guidance can be exploited as part of an interactive design process: live gradient vector visualization for geometric design problems, and taking discrete steps in the design space based on performance information. These strategies are first demonstrated on a simple truss, before being tested on parametric models for a courtyard building and a long-span airport terminal. Initial results show that with further work, gradient-based guidance during interactive design exploration could be a viable strategy for designers seeking more control than is allowed by automated optimization algorithms.

Keywords: computational design, optimization, performance-based guidance, gradient, designer control

1. Introduction

Architectural and structural designers have become increasingly comfortable with using optimization workflows in early design, even for multi-objective problems. In this area, many designers use heuristic methods such as genetic algorithms that are natively available within parametric modeling software. However, the most common workflow of setting up a parametric model, clicking a button to run an optimization, and then returning later to find a single solution has considerable limitations in early design settings. Users miss many potential solutions, have no control over intermediate steps in the process, and gain little context or intuition concerning the final solution that is determined by the computer. This approach to optimization is essentially passive, unless it is repeated with different starting points, conditions, objective priorities, or algorithm settings.

Recently, evolutionary methods have been extended to allow for increased user interaction, which results in more effective preference expression during parametric design (Danhaive and Mueller [1]). However, the development of interactive, performance-based design tools has largely ignored the class of optimization algorithms which involve calculating or estimating the gradient of the objective space. In many formal optimization procedures, the objective space gradient is calculated repeatedly, and this information is used to dictate the next step of an automated algorithm. The gradient shows how objective functions are changing with respect to design variables at a given instance, and thus is used to move towards an optimal solution, which is the general goal of an automated optimization process.

In contrast to full optimization, gradient-based guidance can also be utilized interactively by a designer, rather than as part of an automated procedure. This paper explores two methods for interacting with gradient information during design: live vector visualization and design space stepping. In the first method, a vector indicating the magnitude and direction of the gradient with

respect to each variable and objective is visually projected into the modeling environment itself. Such visualizations have previously been implemented in computer graphics and specific design tools. However, they could also be a useful generalized strategy for multi-objective architectural problems, especially for models with mostly geometric variables and objective functions that can be calculated or estimated in real-time, since they can be explored interactively through continuous visualization. The second method uses finite differences to take individual steps in the objective space that attempt to improve performance or hold it constant, based on the direction of the gradient. While direct control of an objective “slider” is not necessarily possible, this stepping strategy enables design exploration in which designers can improve performance while constantly adjusting step size, switching objectives, interrogating tradeoffs, and shifting regions of the design space by finding isoperforming solutions.

Although live vector visualization and design space stepping have similar requirements to other gradient-based or interactive methods, including continuous or ordered design variables and fast or approximated performance simulations, these methods offer an alternative to passive optimization workflows by giving designers more control. As such, gradient-based guidance is well suited for early geometric design problems with multiple performance objectives and potential design tradeoffs.

2. Literature review

Many optimization algorithms exist for addressing design problems that have multiple or competing objectives, which is often the case even in early stages of architectural or structural design. Common heuristic approaches include simulated annealing, particle swarm, and evolutionary algorithms (Deb [2]). Other methods include Normal Boundary Intersection (Das and Dennis [3]) and weighted sum methods (Marler and Arora [4]), which can involve gradient-based search. Numerous optimization algorithms involving gradient information also exist, including line search methods, conjugate gradient methods, and quasi-Newton methods (Nocedal and Wright [5]). Heuristic and gradient methods have been hybridized in different ways (Bosman and Jong [6]; Goh *et al.* [7]). A review of recent applications in building design indicates the popularity of heuristic methods over gradient-based approaches (Evins [8]), especially when user interaction is involved, leaving the opportunity to apply gradient approaches in new ways.

The need for interactive visualization and other techniques for supporting decision-making during multi-objective conceptual design has been addressed in various fields, including groundwater monitoring (Kollat and Reed [9]), aircraft design (Sun [10]), and product design (Baril *et al.* [11]). Various tools have been created for interactive visualization in engineering, such as RAVE [12] and CityPlot [13]. Similarly, the concept of isoperformance has been used extensively in other fields, including de Weck and Jones [14] in aerospace engineering. In architecture, multi-criteria visualization tools such as Design Explorer by Thornton Tomasetti, which relies on precomputation and uploading solutions to an interactive database, and Octopus [15], which implements multi-objective optimization inside Grasshopper, have recently gained popularity.

Other research specifically involving gradient visualization and stepping through the design space is also relevant. The direct visualization of gradients on top of geometric models has been employed by Whiting [16] to direct architects towards feasible masonry solutions, and by Tacit.Blue (Burnell [17]), an app for 2D design problems presented to the authors during an MIT course. Similarly, interactively and iteratively exploring specific regions of the design space has been demonstrated in a variety of engineering disciplines (Tappeta and Renaud [18]). Stepping through the design space based on separate performance indicators has been proposed by Kesik and Stern [19] for the design of passive solar houses. Michalek and Papalambros [20] offer a system for optimizing architectural layouts interactively, while adding, deleting, or modifying variables, objectives, and constraints.

However, the literature does not contain implementations of gradient-based guidance for interactive conceptual design that are generalized to any simulation or geometry during parametric design. This paper proposes this new application and demonstrates an implementation within Grasshopper for interactive gradient-based design exploration. Performance-conscious architects are thus able to use these methods, but they are also available to structural engineers, mechanical engineers, daylighting experts, energy modelers, or any other designers familiar with parametric workflows.

3. Methodology

This section describes the procedure for implementing live gradient visualization and gradient-based design space stepping within parametric design. The mathematics behind gradients are first described, and then a simple 2D truss example demonstrates how these techniques are applied during early design exploration involving performance objectives as design goals.

The ability to provide gradient-based guidance requires the calculation or estimation of the gradients for each objective under consideration. Directions of objective improvement or isoperformance follow from this initial gradient calculation. For the applications in architectural or structural design envisioned by this paper, which include interactive multi-objective exploration, analytical solutions are rarely possible. In order to realize this methodology as a general approach within parametric design for any type of simulation, a finite central differences approximation for the gradient is used:

$$\frac{\partial J_{z,1}}{\partial x_n} \approx \frac{J(x+\frac{1}{2}h) - J(x-\frac{1}{2}h)}{h} \quad (1)$$

In the above equation, $J_{z,1}(x) \dots J_{z,m}(x)$ represent all objective functions for a design, while $x_1 \dots x_n$ represent the design space variables, collectively called the design vector. The objective functions can involve any calculation or simulation that generates a quantity that should be minimized, maximized, or targeted during design exploration. Using this finite differences approximation, the gradient of the objective function is given as:

$$\nabla J_{z,1} = \begin{bmatrix} \frac{\partial J_{z,1}}{\partial x_1} \\ \vdots \\ \frac{\partial J_{z,1}}{\partial x_n} \end{bmatrix} \quad (2)$$

The gradient at a given point is the direction of maximum rate of change of a function at that point. Thus, moving through the design space either opposite the direction of the gradient (when minimizing) or in the direction of the gradient (when maximizing) will in most cases improve the performance of the design. For multi-objective problems, the system Jacobian must be calculated, which is formed by the gradients of each separate objective:

$$\nabla J_{z,m} = \begin{bmatrix} \frac{\partial J_{z,1}}{\partial x_1} & \dots & \frac{\partial J_{z,m}}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial J_{z,1}}{\partial x_n} & \dots & \frac{\partial J_{z,m}}{\partial x_n} \end{bmatrix} \quad (3)$$

From the Jacobian, it is also possible to complete a singular decomposition and calculate the null space of the Jacobian:

$$U \Sigma V^T = \nabla J_z^T \quad (4)$$

$$V = [v_1 \dots v_m \quad v_{m+1} \dots v_n] \quad (5)$$

In these equations, $v_1 \dots v_m$ form the column space, and $v_{m+1} \dots v_n$ form the null space. From [14], the null space of the Jacobian for a multi-objective problem contains $n - m$ vectors, and any linear combination of these vectors points in a performance invariant direction. For the specific case with only two design variables, it is also possible to move in a direction orthogonal to the gradient in search of designs that do not change performance, since only two of these directions exist.

As described in the introduction, designers can use the information provided by the gradient in two ways. First, the gradient can be calculated and then projected into the design environment, especially with respect to geometric variables. An example of this technique is illustrated in Figure 1. When a geometric design variable describes the location of a node for a structure or building, a visual vector is projected originating from that node, pointing in the direction that the node should move in order to improve performance. The lengths of the vectors represent the relative magnitudes of how quickly the objective is changing at that point, with a longer vector showing that movement in that direction can make performance much better than for a shorter vector. Although the provided example is static, this

gradient information is ideally dynamic, so that designers can adjust variables in real-time and gain feedback on how the objective functions change throughout the design space.

The second design approach involves using gradient information interactively to take discrete steps through the design space, while exploring both quantitative and qualitative design implications. This method is similar to optimization algorithms, but allows designers the control and flexibility to switch or add objectives, change step sizes, or otherwise modify the design at any point, all while visualizing how the design is morphing and continually assessing its quality in ways that are not reducible to objective functions. The authors have implemented this approach as a compiled C# component in Grasshopper, which was used to generate results for the design case studies in this paper. The component, called Stepper, allows a designer to find directions of improving performance or isoperformance for any parametric model and any objective functions modeled within the Rhino/Grasshopper environment.

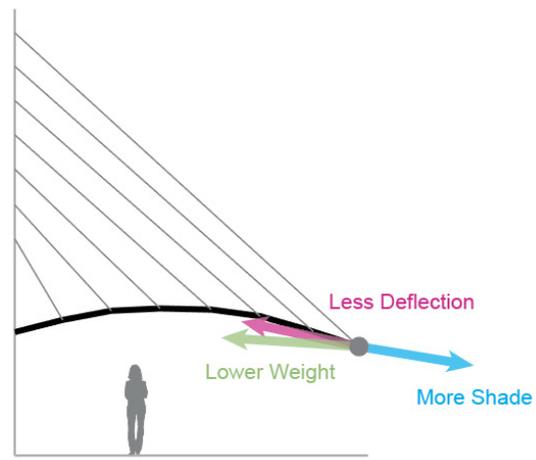


Figure 1: An example of live gradient vector visualization for moving a canopy edge during conceptual design exploration.

Stepper asks the user to provide variable sliders, which dictate the values and bounds of the design vector, as well as a list of objectives and an objective index, step size, and direction for moving through the design space. The direction can be either negative, positive, or zero. When activated, Stepper will first calculate the gradient through finite differences. The component then takes one step in the direction of the gradient (or its opposite, if direction is negative) by changing each variable slider value from (x_n) to $(x_n + \frac{\partial J_{z,m}}{\partial x_n} * h)$, using the step size normalized by each variable range as h . If isoperformance is desired and the provided direction is zero, Stepper calculates the null space of the system Jacobian and selects a direction to move based on these vectors. In the two-variable case, Stepper moves either left or right with respect to the gradient. For higher dimensional problems, selecting appropriate null space vectors is more complicated, and will be discussed in Section 5.

To better visualize how these gradient-based design strategies are applied in conceptual design, consider the simple seven bar truss illustrated in Figures 2-3. The variables for the problem dictate the height and width of the truss, while the objective is to minimize its weight while carrying a single point load at the center. The simulations for the truss were completed using the Optimize Cross Section feature within Karamba. For this seven bar truss, the right side of Figure 2 shows steps taken away from an initial design based on the estimated gradient, either in the direction of improving performance or along what are intended to be isoperformance contours. The paths through the design space are visualized on top of the contours of the objective space and its optimum, which were approximated prior to initiating exploration. Since the sizing algorithm leads to a discontinuous design space, approximation techniques do not always lead to the desired solution, especially if inappropriate step sizes are chosen. However, by allowing the user to interact with the design process, he or she is able to continuously test, adjust, consider, and move, while gaining a better understanding of the design space and ultimately arriving at an acceptable solution.

The left side of Figure 3 zooms in on a single step of the truss problem, depicting the level of information that can be provided to a designer at this point. Once the gradient is estimated, the designer can see a visualization of potential steps in directions of improving performance or static performance. While moving farther away from the initial point reduces the likelihood that additional steps in that direction will continue to influence objective functions in the same way, this information can all be helpful to understanding the design space. While stepping around to different designs, the local shape of the objective space and its gradient changes, as demonstrated by the actual simulations

depicted as blue dots on the right side of Figure 3. In this graphic, steps in any direction closely match the expected behavior, either improving or worsening along with the gradient, or remaining flat when moving in an estimated performance-invariant direction. This relationship fades when moving away from the initial point—however, an interactive visualization can be updated with each step, gradually informing the designer of where best to move and how performance is likely to change while exploring the design space. These concepts are tested further in the following case studies, which consider performance-drive designs related to energy, structures, and daylighting. In these examples, objective function results are shown as a percent change from a baseline performance.

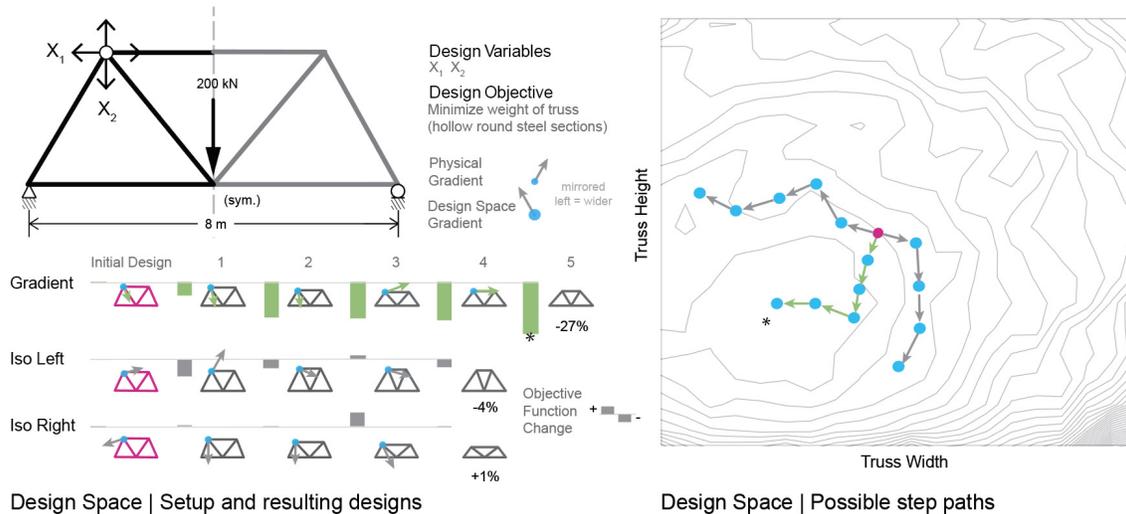


Figure 2: Potential steps in the design space based on gradient information about how the design is performing

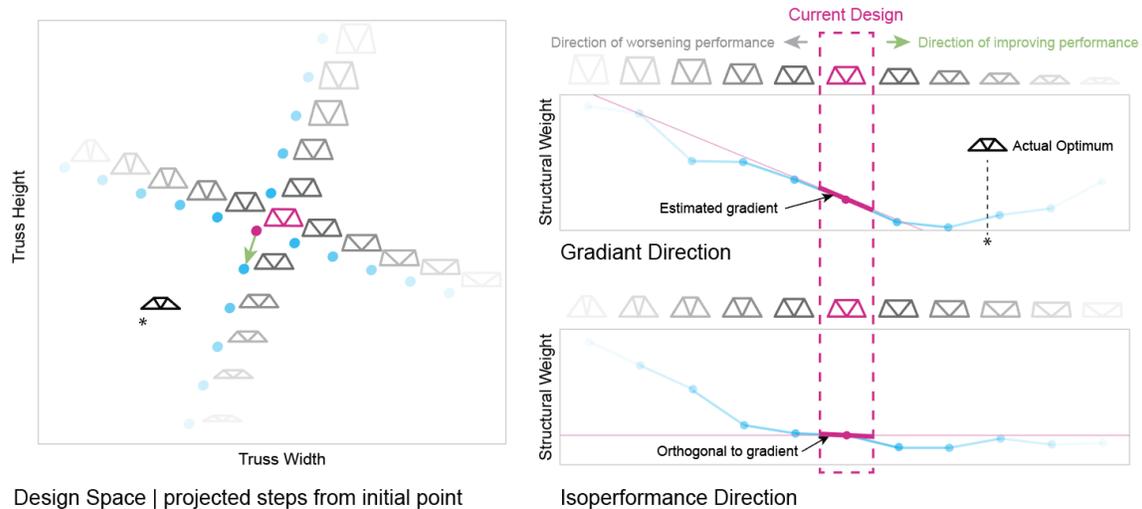


Figure 3: A visualization of projected outcomes for a single step in the design space

4. Case study: gradient stepping for a courtyard building design

The first case study involves a conceptual massing design for a rectangular courtyard building. This problem relates to early massing studies regularly completed by architects when developing building forms across a variety of scales, locations, and uses. The building is three stories tall, with a footprint of 30 x 40 meters. The design variables are the corner heights of both the interior and exterior facades, which can move both up and down. The objectives are the PV potential of the roof geometry, which should be maximized, as well as the lighting, heating, and cooling loads calculated for the geometry, which should be minimized. Together, the combined loads minus the PV potential represent the total

energy requirements that depend on the building massing, which neglects hot water, equipment, and any other loads. Despite containing one overall objective, it is still useful to give designers control of these separate loads in the conceptual stage—for example, designers might have specific non-geometric mitigation strategies for certain loads that will be added in a later design phases, and want to first focus on lowering certain impacts through geometry. The unmodified form closely resembles a standard building used for building physics simulations, but geometric complexity is added when manipulating the design variables, since this leads to different pitches and even curvature on the roof.

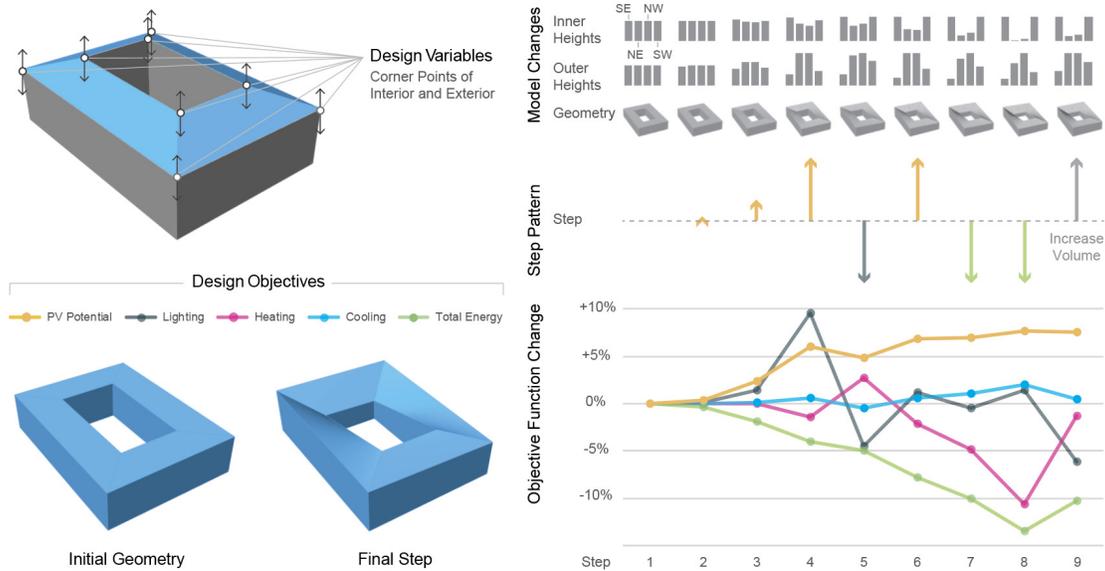


Figure 4: A potential path through the design space based on steps taken for different performance objectives

The geometry of the building was modeled in Grasshopper, and each of the PV and energy simulations were completed using the analysis plug-in DIVA. The massing is broken into three floors and four zones for each floor for the energy models. A Boston weather file was used to complete the annual energy simulations. In order to achieve full interactivity for exploration, a surrogate model of each load was constructed using the Tilde component within the Design Space Exploration suite of tools. This component took in 200 simulations that were precomputed and generated an Ensemble Neural Network model for each load. Upon completion of this case study, every design considered was re-simulated to ensure that the surrogate models provided acceptable accuracy for design exploration. The vast majority of surrogate values used were less than 3% from a simulated value, which is essentially within the error of the energy simulations themselves and considered reasonable for an early design study comparing the relative performance of different options.

Figure 4 demonstrates one possible design space stepping history for the problem created by the authors, out of essentially infinite possibilities. This example begins with a basic flat roof geometry, establishing a baseline energy usage. At first, steps are attempted to increase the PV potential of the building, in different sizes to gain an understanding of what step size leads to a worthwhile design response. After taking three steps to improve PV, the designer experiments with different objectives to understand how the geometry changes. Realizing that most steps are reducing the overall building volume, which may be cutting into usable floor area in some third floor locations, the designers decided to add volume as an objective and step in a larger direction to test sensitivity. Any design in this history may be worth considering, and at any point, a different direction could be taken to explore another interesting area of the design space. In each step, the objective functions provided the desired response, essentially giving the user an ability to control the design by adjusting objectives rather than variables, which is a potentially powerful approach to performance-driven design.

5. Case study 2: isoperformance stepping for an airport terminal design

The second case study explores tradeoffs between daylight and structural efficiency for a long span roof design inspired by the SFO International Terminal in San Francisco, CA. While the last case study focused on stepping with the gradient, this section considers stepping for isoperformance. The design problem has 11 variables related to the global geometry that affect both structural form and availability of interior daylight. The structural objective is to minimize steel weight subject to a 4.79 kN/m² vertical load and 1.44 kN/m² lateral load, again calculated using the sizing algorithm of Karamba, and the daylight objective is to maximize Spatial Daylight Autonomy (sDA), simulated by DIVA assuming a generic white interior and translucent panels for all skylights and clerestory glazing. As with the last case study, a surrogate model of each performance objective was created and used for analysis. The 11 variables allow for testing two effects related to the curse of dimensionality that arise with this methodology. The first is that steps that are technically similar in size to those in lower dimensional problems may lead to differences that are less visually and geometrically meaningful, since the distance travelled in each variable direction contributes to the total overall step length. The second issue is that many null space vectors calculated from the Jacobian can point almost entirely in the direction of a single variable, which is not necessarily a compelling direction for exploration.

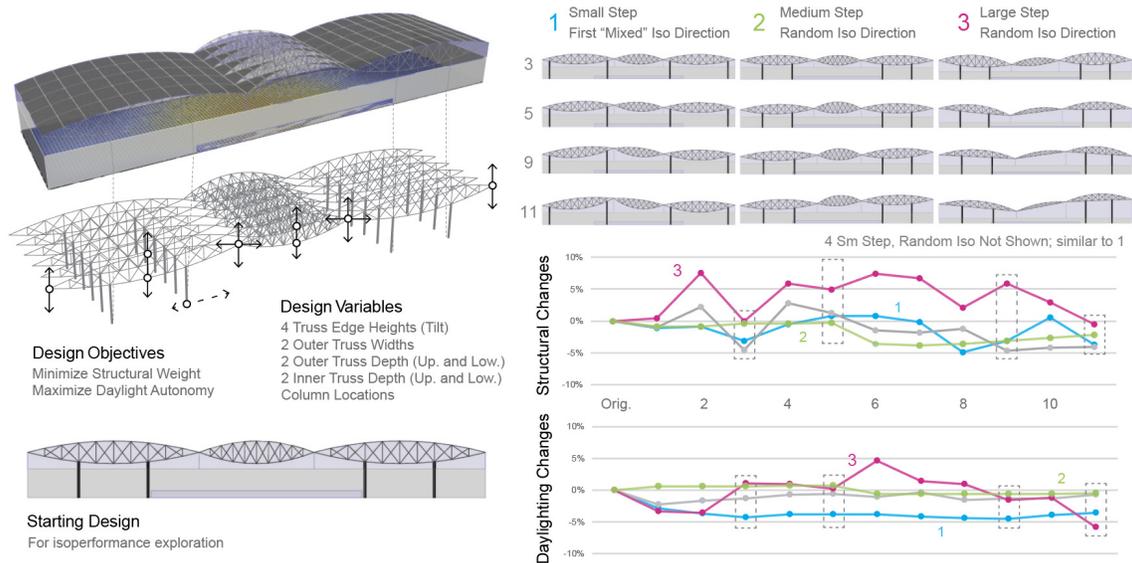


Figure 5: Attempted isoperformance exploration using different approaches for step size and choosing directions

Both issues relate to the probable purpose of isoperformance design exploration: taking large enough steps that the resulting designs are meaningfully different, while ideally maintaining similar performance. Consequently, this paper tests the design response of various step sizes, as well as different strategies for mitigating the null space vector issue. The exploration step sizes represent faster and slower paths through the design space. For the selection of isoperformance directions, three possibilities are tested: randomly selected vectors, cycling through the null space vectors and choosing the first vector for which the difference between the first and second variable changes is above a provided threshold, and a hybrid of the two. The histories of example isoperformance stepping paths are provided in Figure 5. Even on initial consideration, the calculated design space directions do provide visually diverse possibilities for the roof geometry. However, the ability of this method to find designs with constant performance is mixed, as some steps result in larger performance changes than is likely desirable. In the specific case of the largest step size, the surrogate model likely does not perfectly predict the actual performance in this region, as the shallow central truss should have a much larger structural performance penalty. Although designers could pick different step sizes to suit their own thresholds for visual and objective difference, it is clear that further experimentation is required to pick best default strategies for an effective isoperformance design tool.

6. Future work and conclusions

This study has a number of areas for future work. Although the authors have created an initial Grasshopper tool for design space stepping, many visualizations in this paper were created manually. Future iterations should integrate visualizations of potential design outcomes and performance histories directly into the design environment. In addition, although surrogate modeling drastically improves the speed of the gradient approximation within Grasshopper, the stepping component still requires some time to refresh solutions and output results. Thus, the whole process needs to become slightly faster to achieve true real-time feedback without hesitation. In addition, the isoperformance stepping strategies for higher dimension problems require further exploration and testing before this functionality can be a robust design strategy. It is also worth noting the magnitudes of possible errors introduced into the simulations by attempts to make this workflow interactive. For the case studies, changes in performance were often within 5-10%, which may be in the range of error for the initial simulations, especially when using surrogate modeling. However, such modeling techniques can be useful for early stage design and creative exploration of form, especially when they use relative simulations between alternatives within a single design space to rank these alternatives rather than calculate their absolute performance. In these cases, and especially for multi-objective geometric problems in which the designer seeks to balance both qualitative and quantitative design goals, gradient-based interactive strategies can provide a useful alternative to passive optimization routines.

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