Proceedings of the IASS Symposium 2018
Creativity in Structural Design
July 16-20, 2018, MIT, Boston, USA
Caitlin Mueller, Sigrid Adriaenssens (eds.)

Computational Structural Design and Fabrication of Hollow-Core Concrete Beams

Mohamed A. ISMAIL *, Caitlin T. MUELLER[a]

*[a]Massachusetts Institute of Technology
77 Massachusetts Ave, Cambridge, MA, USA 02139
*ismail@mit.edu

Abstract

The paper presents the results of the design method for a simply supported cavity beam, along with fabrication and load testing results. An optimization algorithm determines the location and rotation of empty plastic water bottles within a prismatic reinforced concrete beam in order to reduce material usage without reducing strength. Designed for India's affordable housing construction, the beam is constrained by the fabrication methods and materials available to India's construction industry. This is an effort to merge structural design tools with the development of affordable housing technology, potentially reducing the economic and environmental cost of construction through material efficiency. The designed beam results in a theoretical concrete volume reduction of 16%. Two cavity beams are designed and constructed, and then load-tested in comparison to two solid beams with the same dimensions.

Keywords: Structural optimization, reinforced concrete, developing economies

1. Introduction

In India, steel-reinforced concrete frames dominate the skylines of rapidly growing cities. In Less Economically Developed Countries (LEDCs), such as India, it is estimated that material costs can constitute 60 to 80 percent of the total cost of building construction (Figure 1) [1] [2]. Nonetheless, construction in these regions mimics the materially inefficient practices of More Economically Developed Countries (MEDCs) that were developed to reduce labor over material costs. As a result, prismatic beams and flat slabs are commonly used despite their structural inefficiency.

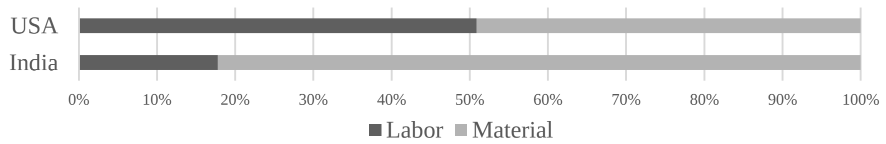
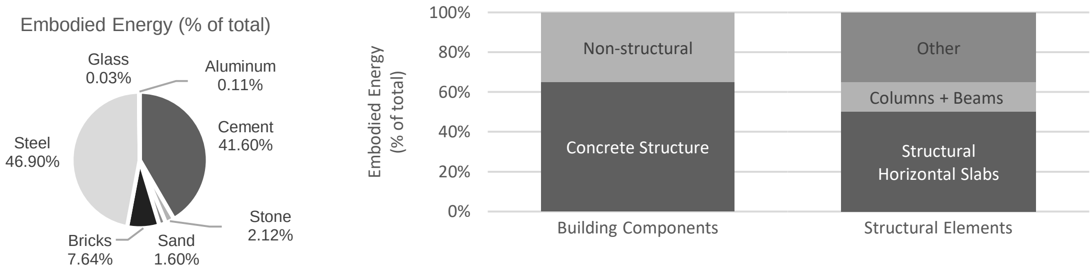


Figure 1: Comparative Costs of Material and Labor for Construction [1] [2]

With global efforts to reduce the lifetime operational energy of buildings through energy efficiency and passive design solutions, it is increasingly necessary to address the remaining embodied energy of buildings. The mounting use of concrete structures in India's cities has led to a developing concern for the environmental costs of construction; India's construction sector accounts for 22 percent of India's carbon emissions [3]. The impact of these materials on a building's embodied energy are immediately apparent: cement and steel are responsible for nearly 90 percent of a multistory concrete frame building's total embodied energy [4] and approximately 50 percent of that is in the horizontally-spanning elements alone (Figure 2) [5].

Therefore, there is an opportunity for materially-efficient structural elements that can reduce the economic and environmental costs of construction. Pairing accessible fabrication methods with computational structural optimization, this research explores the material optimization of concrete horizontal spanning components. Designed for the construction constraints of India, the structural elements are optimized to reduce the necessary volume of concrete while resisting the same loads of an equivalent solid rectangular prismatic beam or slab.



a b

Figure 2: Embodied energy breakdown of a) multi-story Indian house [4] b) multi-story concrete building [5]

2. Literature Review

2.1. Structural design in India

Historically, designers have worked towards material efficiency by developing novel solutions never before seen in India. In his design for the India International Center in New Delhi (1962), Joseph Allen Stein utilized ferrocement channels and modular blocks to reduce the amount of concrete in long-spanning roof and floor systems for public spaces and offices (Figure 3a). The architect Laurie Baker pioneered techniques for low-cost housing design in India, constructing homes for one-fifth the cost of competing designers. His techniques included filler slabs and vaulted systems that utilized clay brick and reduced the need for steel reinforcing. In the 1950s through 60s, Indian architects and engineers such as B. V. Doshi and Mahendra Raj introduced novel systems of efficient and long-spanning concrete systems such as thin shells, folded-plates, and space frames while seeking a modern vernacular for the newly independent nation.

a b c

Figure 3: a) IIC, New Delhi, 1958, Joseph A. Stein, b) AVEI, ferrocement channels, c) TARA, filler slab

Today, architects and researchers in India are continuing the legacy of these designers. The construction of TARA's recent headquarters in New Delhi and at the Auroville Earth Institute (AVEI) near Pondicherry utilized similar techniques to those discussed above (Figure 3b, c). Despite these efforts, much of the ongoing research in India is now focused on the material configuration of construction, using waste products or local resources as building materials, and prefabricated systems that are easily assembled yet reliant on imported materials and high transportation costs [6].

2.2. Structural design beyond India

Many of the historic innovations in India's long-spanning concrete systems mirrored the materially efficient construction being done in Europe and the USA at the same time. At the time, material constrained construction costs in many parts of the world, while labor was relatively inexpensive and available. These innovations included, among others, the thin concrete shells of Heinz Isler and Felix Candela, the monumental frames of Pier Luigi Nervi, and the long-spanning vaults of Eduardo Torroja. Once the cost of labor in MEDCs became the controlling factor in building construction, the interest in materially-efficient structures gave way to easily erected, modular systems.

Work is once again being done to develop efficient horizontal-spanning structures, but the focus is now on the reduction of embodied energy rather than cost of construction. Newly developed systems utilize digital-fabrication and prefabricated elements that require little to no human labor and relatively complex methods of production. Researchers at ETH Zurich developed a number of floor systems that are primarily compressive, reducing the need for steel reinforcement [7]. Designers have also developed numerous forms for fabric-formed concrete – elements shaped according to the presumed flow of forces and fabricated using light-weight formwork [8]. Researchers at the University of Cambridge are exploring lightweight alternatives to two-way concrete slabs that resemble the vaulted floor systems of historic masonry construction [9]. New tools in digital optimization and structural analysis have bolstered the design of materially-efficient structures by allowing for increased precision and predictability in structural design.

2.3. Research aims

The solutions developed in India have had the advantage of being designed and tested within their setting, yet they remained isolated case studies. Little study and understanding of their behavior still leaves their design to rules-of-thumb and additional factors of safety out of precaution. Additionally, new precedents in floor system design rely on high-technology fabrication methods and complex procedures, limiting their use to very few regions of the world today. This research bridges the gap between developments in structural systems within India, and digital design and fabrication methods.

3. Methodology

3.1. Conceptual overview

The research presented here involves a methodology of four components. First, a conceptual design for the beam is developed using visual programming and digital structural analysis. Second, a series of physical tests explores both the appropriate mix design and the fabrication methods appropriate to the design proposal. Third, the design is revised according to the previous step and the test beams are fabricated. Lastly, the optimized beams are load tested and compared to control beams.

3.2. Computational design and analysis

The design and analysis of a control prismatic beam is prepared according to common design practice. This initial design references the reinforced concrete design procedure outlined in the American Concrete Institute's ACI 318-14, also referred to as ultimate strength design, LRFD, or ULS. This method assumes that concrete performs in compression near the beam's ultimate strength, while tensile stresses are handled by longitudinal reinforcing steel.

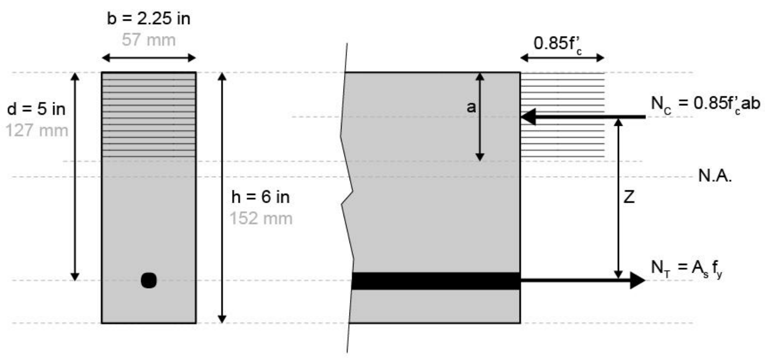


Figure 4: Ultimate strength design method outlined by ACI 318

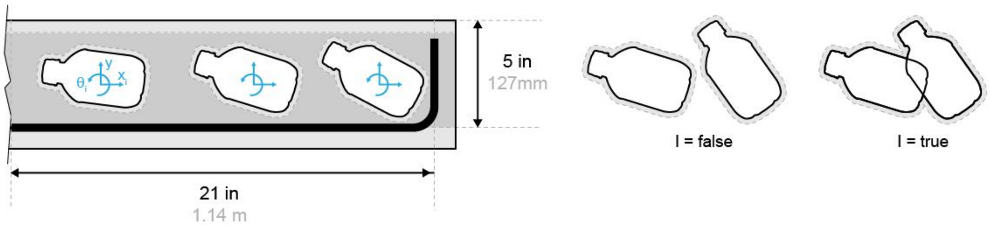
The tensile and compressive stresses are equated using the Whitney stress block assumption shown in Figure 4, and the moment capacity is determined using the moment arm between the compressive block and tensile steel as shown in equation (1):

$$\boldsymbol{M_N = N_C Z \text{ or } N_T Z} \tag{1}$$

The control beam has the cross section shown in Figure 4 and an unsupported span of 42in (1.07m). The longitudinal reinforcing consists of one #3 reinforcing bar of steel with a yield stress of 60ksi (414MPa), 0.325in (8.26mm) in diameter chosen for fabrication ease. According to ACI code, the simply-supported beam should resist a 2.8kip (12.5kN) concentrated load at mid-span.

The cavity beam is designed with voids cast around empty plastic water bottles as "lost formwork". Bottles are chosen as a viable enclosed volume, but other waste products may be substituted. The design is realized through digital shape optimization and structural analysis, using the 3D modelling software Rhinoceros 5.0, the visual programmer Grasshopper for Rhino5, and the Grasshopper plugins Karamba 3D for structural analysis and the optimization solver, Goat [10]. A Grasshopper definition locates distinct bottle silhouettes inside a representative model of the control beam. Then, the cavity beam is analyzed using Karamba 3D's finite element analysis and assessed for structural utilization and total strain energy.

There are three discrete parameters defined for optimization: the type of bottle, its location within the beam, and its rotation about its centroid. The optimization problem minimizes strain energy and volume by manipulating these three parameter. Additionally, a number of constraints are placed on the solver as explained in equations (2) through (7). To reduce processing times while assuming symmetrical loading, the bottles are located symmetrically about the mid-span axis and aligned vertically at the same depth. An initial optimization is run using Goat's local, quadratic algorithm – appropriate for smooth and continuous problems – in order to minimize the strain energy by manipulating the bottles' locations and rotations. A second optimization is run using Goat's global, deterministic algorithm – suitable for a combinatorial problem – to minimize the beam's volume by changing the type of bottles. The utilization problems are defined below.



Symbol	Variable	Range
$\boldsymbol{x_i}$	horizontal location of bottle *i*	$0 < x < 21$ in (533 mm)
$\boldsymbol{y}$	vertical location of all bottles	$0 < y < 5$ in (127 mm)
$\boldsymbol{\theta_i}$	rotation of bottle *i*	$-{}^{\pi}/_{2} < \theta < {}^{\pi}/_{2}$ rad

Figure 5: Bottle optimization parameters and constraints, light grey areas result in penalty when crossed

$$\boldsymbol{x} = \begin{bmatrix} x_i \\ y \\ \theta_i \end{bmatrix} \tag{2}$$

$$\min W_{IN}(\boldsymbol{x})\big(1 + P(\boldsymbol{x})\big) \tag{3}$$

Here, $\boldsymbol{x}$ is the design vector containing all design variables, W_{IN} is the internal strain energy of the beam as calculated by Karamba 3D, and $P(\boldsymbol{x})$ is the penalty jump function defined below.

$$P(\boldsymbol{x}) = \begin{cases} 10^6 \text{ if } I = \text{true} \\ 0 \text{ if } I = \text{false} \end{cases} \tag{4}$$

Where I is the boolean expression for object intersection defined in Figure 5 above.

$$\min V(\boldsymbol{x})\,\big(1 + P_1(\boldsymbol{x}) + P_2(\boldsymbol{x})\big) \tag{5}$$

Here, $V(\boldsymbol{x})$ is the total volume of the beam estimated by Grasshopper, and $P_1(\boldsymbol{x})$ and $P_2(\boldsymbol{x})$ are the penalty jump functions defined below.

$$P_1(\boldsymbol{x}) = \begin{cases} 10^6 \text{ if } \Delta_{max} \geq \frac{L}{360} \\ 0 \text{ if } \Delta_{max} < \frac{L}{360} \end{cases} \tag{6}$$

$$P_2(\boldsymbol{x}) = \begin{cases} 10^6 \text{ if Utilization} > 100\% \\ 0 \text{ if Utilization} \leq 100\% \end{cases} \quad (7)$$

The variable Δ_{max} is the maximum deflection of the beam as calculated in Karamba 3D.

3.3. Concrete mix design

A custom mix design is needed to maintain workability without forfeiting the structural capacity of the concrete. The mix is designed with reference to the ACI 211.1 code, using a recipe of cement, water, fine aggregate, and chemical plasticizer for increased flow shown in Table 1. Coarse aggregate is not used due to the small scale and relative complexity of the formwork. The mix is designed for a 28-day compressive strength of 4ksi (27.6MPa) in compression. To verify the strength of the mix, 3in (76.2mm) by 6in (152mm) cylinders are cast for compressive testing from the mixes that the beams are cast from. To assure workability, a slump test is conducted for each mix and the results are included in Table 1.

Table 1: Concrete mix design and final quantities

	Design (%)	**Mix 1**	**Mix 2**	**Source**
Water, lbs (kg)	12%	11.9 (5.4)	13.2 (6.0)	
Cement, lbs (kg)	21%	20.8 (9.4)	23.0 (10.4)	Lehigh Portland Cement
Sand, lbs (kg)	68%	68.6 (31.1)	75.5 (34.2)	Ace-Crete Special Sand
Plasticizer, oz (ml)	9 oz/100 lbs	10.0 (296)	11.0 (325)	GCP Adva-Cast 575
Slump, in (mm)		1.0 (25.4)	1.5 (38.1)	

3.4. Formwork and casting

A 0.75in (19.1mm) plywood form is used to fabricate each of the control beams, and a variation of this formwork is used for the cavity beams. To accurately locate the bottles within the formwork, impressions of the bottles' surfaces are milled into rigid insulation sheathing panels using a three-axis CNC router. The bottles are then adhered to the milled impressions, holding the insulation panels together while the rest of the formwork is constructed from plywood. The width of the beams is controlled by the diameter of the bottles, giving them surface-to-surface contact with the formwork. Longitudinal steel reinforcing is cut down to 5ft (1.52m) segments, and bent to fit the formwork using a manual rebar bender. Due to the small scale of the beams, custom 1in (25.4mm) rebar chairs were 3D printed to hold the longitudinal steel in place. For this paper, a total of four beams were cast: two control beams and two cavity beams.

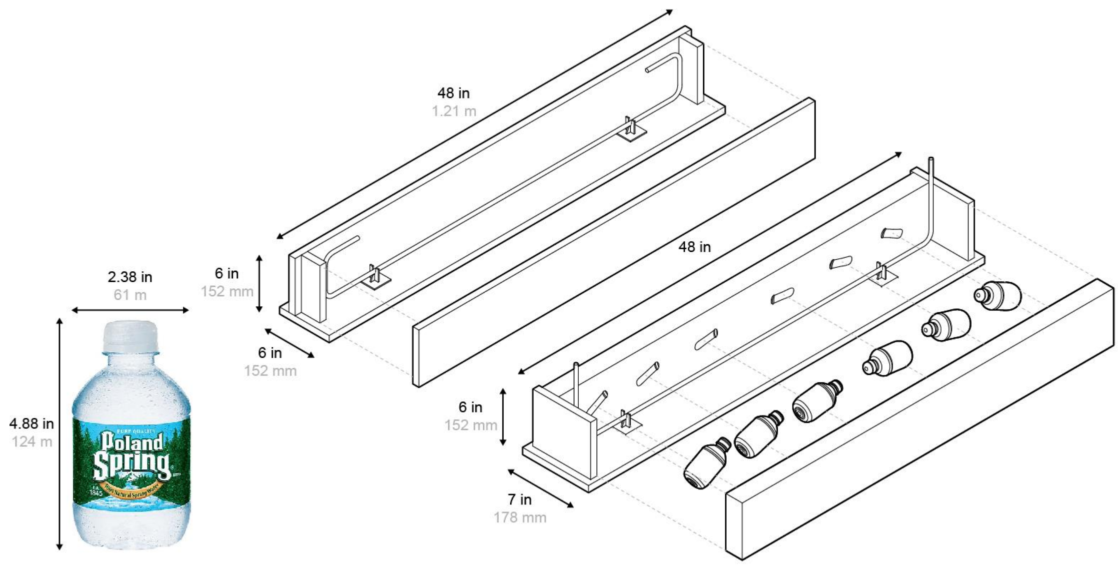


Figure 6: Diagram of formwork construction and bottle dimensions

a b c d

Figure 7: a) CNC milled bottle indents, b) Bottles adhered to form, c) Slump test, d) Cast beams set to cure

3.5. Structural load testing

Each beam is subjected to a loading test with simply supported three-point bending after fourteen days of curing. The unsupported span is 42in (1.07m) with 1.5in (38.1mm) of additional span beyond each support. Load is applied with a central hydraulic jack, and the test is displacement-controlled. Additionally, the cylinders are load-controlled tested for compressive strength.

Figure 8: Load test set-up

4. Results and Interpretation

4.1. Computational design results

The cavity beam design is shown in Figure 9. An initial material saving of 16% is expected. Due to the small scale of the beams, the location and types of bottles are constrained more than they would be at full-scale. For the initial fabrication tests, six Poland Spring 8-ounce water bottles are used in each of the cavity beams.

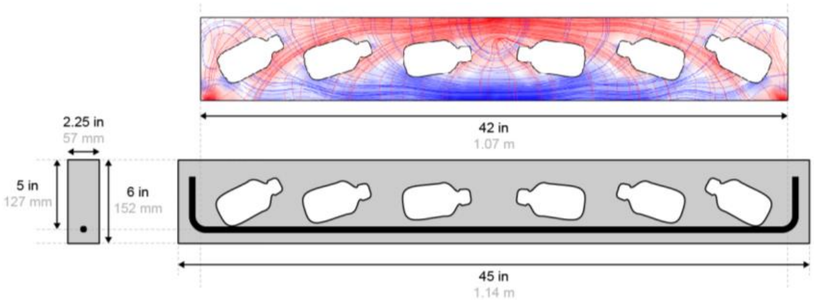


Figure 9: Final design of optimized cavity beams

As shown in Figure 10, a 20ft (6.10m) beam is designed with a depth of 15in (381mm) that illustrates the possible results and consequent material savings of this method at full-scale.

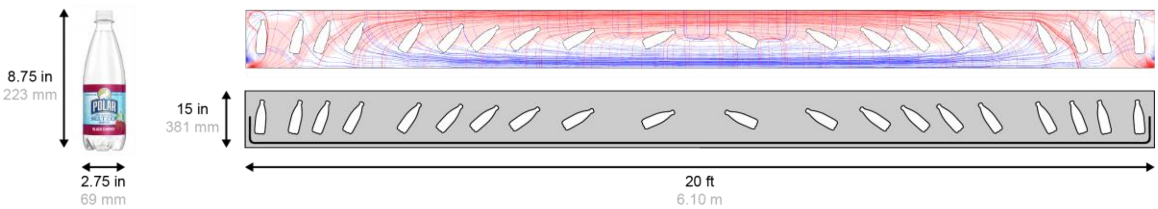


Figure 10: Sample full-scale cavity beam with 20 bottles causing a material reduction of 11%

4.2 Fabrication results

Table 2: Beam labels

Label	Type	Concrete Mix
S_M1	Solid	1
C_M1	Cavity	
S_M2	Solid	2
C_M2	Cavity	

The beams are measured in five locations for height (h) and width (b) and the averaged results are shown in Table 3. Cylinders are measured and weighed to determine a concrete mix density of 127lb/ft^3 (2034kg/m^3). The cavity beams unintentionally expanded in width due to the flexibility of the formwork, resulting in 20-23% additional volume and weight for the cavity beams.

Table 3: Preliminary measurement of beams after fabrication

Beam	Length, in	Width, in	Height, in	Weight, lb	Volume by weight, ft³	Design Volume, ft³	% error
S_M1	45.0	2.32	5.75	44.8	0.350	0.351	**0%**
C_M1	45.1	2.67	5.93	49.0	0.383	0.295	**-23%**
S_M2	45.0	2.34	6.08	47.2	0.372	0.351	**-6%**
C_M2	45.1	2.55	6.10	46.8	0.369	0.295	**-20%**

4.3. Load testing results

Table 4: Cylinder maximum stress

Mix	Stress, psi	Avg. Stress, psi (MPa)
1	3144	3245 (22.4)
	3346	
2	3290	2906 (20.0)
	2550	
	2879	

Table 5: Beam test results, maximum loads

Beam	Weight, lb (kg)	Maximum Load, lbf (kN)	Normalized Load, lbf/lb
S_M1	44.8 (20.3)	3482 (15.5)	77.7
C_M1	49.0 (22.2)	2849 (12.7)	58.1
S_M2	47.2 (21.4)	3207 (14.3)	67.9
C_M2	46.8 (21.2)	2544 (11.3)	54.4

As seen in Table 4, the compressive strength of each mix is less than the designed stress capacity of 4ksi (27.6MPa). Testing occurs only sixteen days after casting due to time constraints, reducing the concrete strength to ~90% of design capacity. The maximum load for each beam is normalized by their weight to account for the increase in volume, and the results are shown in both Table 5 and Figure 11 below.

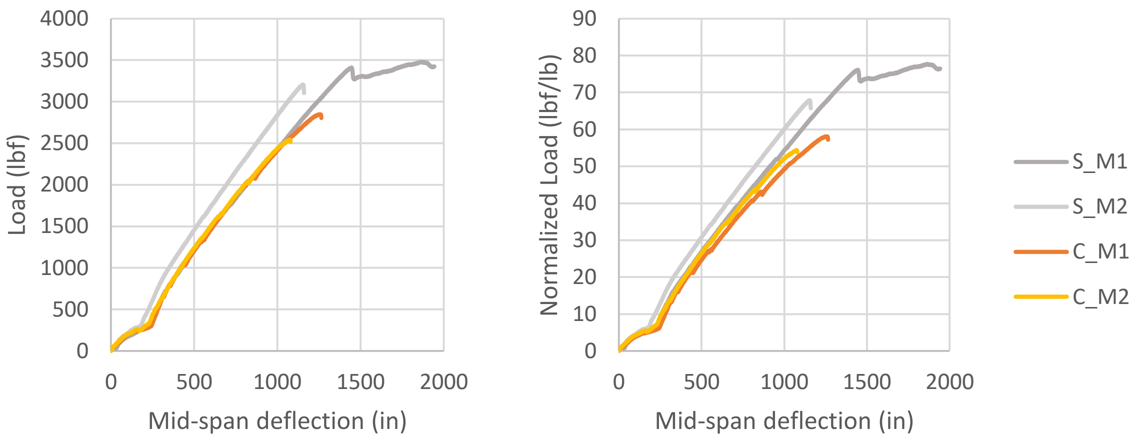


Figure 11: a) Load-displacement plot of beam tests b) Same plot with loads normalized by weight

Beam C_M1 reaches the design load of 2.8kip (12.5kN) while beam C_M2 reaches a peak load of 2.5kip (11.1kN). Other than S_M1 all beams fail abruptly in shear as opposed to the ideal ductile failure. Once normalized for their weights, all four beams behave similarly up until failure, and both cavity beams fail in shear sooner than the control beams possibly due to stress concentrations around the bottles. It can be seen in the following images that the cavity beams both failed with shear cracks forming through the second cavity from the left-hand support. To summarize, the cavity beams could be designed to resist similar loads to solid beams but additional attention must be paid to their shear capacity to allow for increased ductility.

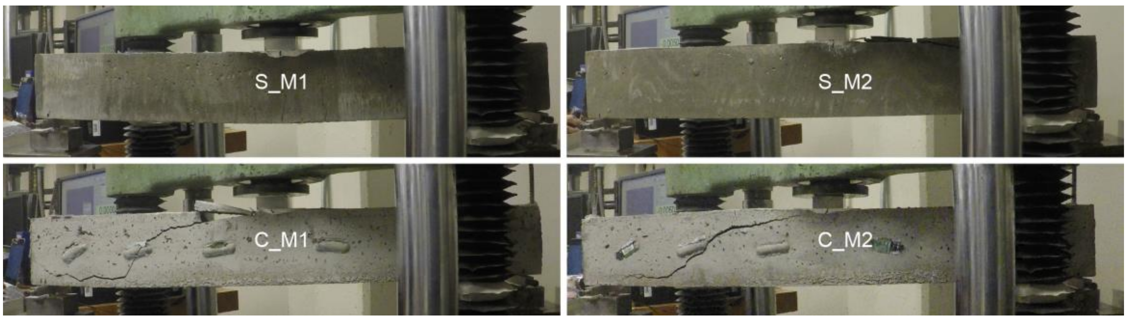


Figure 12: Beam test results; beams S_M2, C_M1, and C_M2 resulted in brittle shear failure

5. Conclusions

5.1. Contributions and future work

Bridging the gap between academic work in structural design, and the global effort to alleviate housing insecurity this research explores an unrealized opportunity to reduce the cost of housing construction through material efficiency. Additionally, the paper presents a methodology of "design, build, test, and design" that could recur in the design of materially-efficient structural elements for housing in LEDCs.

In future work, several limitations of the discussed methodology should be addressed: only one type of bottle is used, bottles have a limited region for translation and rotation due to scale, testing is done after only sixteen days of curing, no coarse aggregate is used, beam width is limited to that of the bottles, formwork is too flexible for accurate casting, and beams have insufficient shear resistance leading to sudden brittle failure. Future work can explore additional fabrication methods that represent the construction possibilities of India. This includes accurate cavity placement within formwork, alternative waste products, designing for shear resistance, pre-fabricated elements that prevent inaccurate casting, and additional sample load testing. While this research focuses on prototype-scale testing, full-scale testing and implementation – where flexure may control design more than shear – should be explored.

5.2. Discussion

Designed for the material constraints of India, this research explores a methodology of structural optimization that uses consumer waste for material displacement inside of a prismatic beam element. The design presented here reduces the volume of concrete needed by 15% to resist a similar load to a solid reinforced concrete beam. Due to fabrication errors, the cavity beams resulted in a larger volume of material than control beam counterparts – yet, once the data is normalized for weight, they exhibited promising load capacity.

Acknowledgements

The authors wish to acknowledge the assistance of undergraduate research assistant, Claire Holley. Additionally, thank you to the Tata Center for Technology and Design at MIT and the Tata Trust for their continued support and guidance.

References

[1] CIDC, "Planning Commission: 11th Five Year Plan," 2012. [Online]. Available: http://www.cidc.in/new/activities7.html. [Accessed: 16-Oct-2017].

[2] J. J. Salama *et al.*, *Reducing the cost of new housing construction in New York City: 2005 update*. New York: Furman Center for Real Estate and Urban Policy : New York University School of Law : Robert F. Wagner Graduate School of Public Service, 2005.

[3] B. V. Venkatarama Reddy and K. S. Jagadish, "Embodied energy of common and alternative building materials and technologies," *Energy Build.*, vol. 35, no. 2, pp. 129–137, Feb. 2003.

[4] S. Bardhan, "Embodied energy analysis of multi-storied residential buildings in urban India," in *WIT Transactions on Ecology and the Environment*, 2011, vol. 143, pp. 411–421.

[5] N. Huberman, D. Pearlmutter, E. Gal, and I. A. Meir, "Optimizing structural roof form for life-cycle energy efficiency," *Energy Build.*, vol. 104, no. Supplement C, pp. 336–349, Oct. 2015.

[6] R. Smith and S. Narayanamurthy, "Prefabrication in Developing Countries: A Case Study of India," *Wood Struct. Symp.*, Sep. 2008.

[7] David López López, Diederik Veenendaal, Masoud Akbarzadeh, and Philippe Block, "Prototype of an ultra-thin, concrete vaulted floor system," in *Proceedings of the IASS-SLTE 2014 Symposium*, 2014.

[8] J. Garbett, A. P. Darby, and T. J. Ibell, "Optimised Beam Design Using Innovative Fabric-Formed Concrete," *Adv. Struct. Eng.*, vol. 13, no. 5, pp. 849–860, Oct. 2010.

[9] W. Hawkins, J. Orr, P. Shepherd, T. Ibell, and J. Bregulla, "Thin-shell textile-reinforced concrete floors for sustainable buildings," 2017.

[10] "Rechenraum e.U. - goat." [Online]. Available: https://www.rechenraum.com/en/goat.html. [Accessed: 06-May-2018].